

Transformation

Transformation is the process by which we can change the shape, Size, position and direction of any object.

Two way of Transformation:

- ❖ Geometric Transformation
- ❖ Co-ordinate Transformation

Geometric Transformation: Object is transformed but co-ordinate is not transformed.

There are five types also:

- ❖ Translation
- ❖ Scaling
- ❖ Rotation
- ❖ Mirror reflection

Translation

An object is displaced a given distance & direction from its original position.

from this figure, you can write that

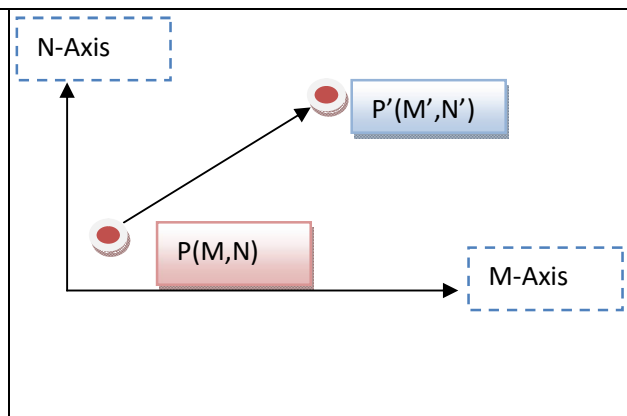
New Point : $P'(M',N')$

Original Point : $P(M,N)$

Where $M'=M+T_M, N'=N+T_N$

We can write it as –

$$\mathbf{P'} = \mathbf{P} + \mathbf{T}$$



Example : In 2-D previous point are required as new point are (12,6) and translation vector is $3i+4j$.

Answer:

New Object : (12,6)

Changes: (3,4)

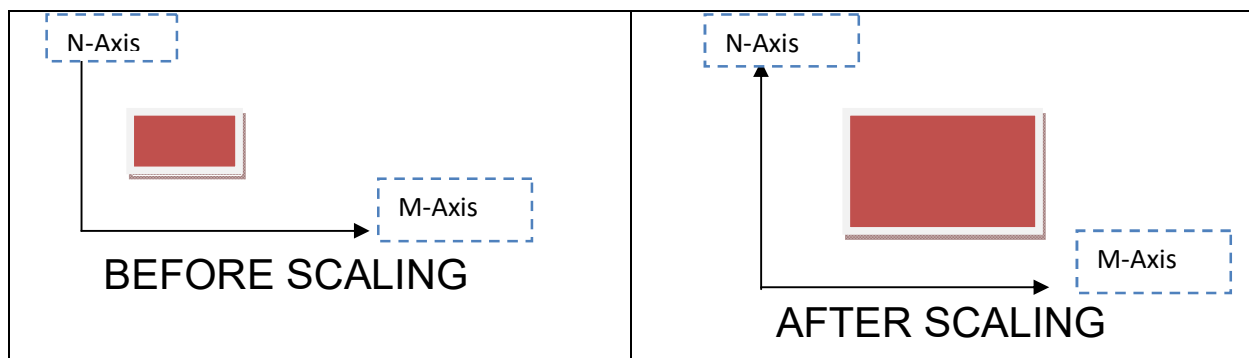
i.e. $M'=12, N'=6, T_M=3, T_N=4$

$M'=M+T_M, M=9$

$N'=N+T_N, N=2$

Scaling

- Process of changing the size.
- Process of expanding or composing of an object.



The original coordinates: M, N

The scaling factors are (S_M, S_N)

The produced coordinates are M', N' .

This can be mathematically represented as –

$$M' = M \cdot S_M \text{ and } N' = N \cdot S_N$$

Example : Given a square object with coordinate points $A(0, 4)$, $B(4, 4)$, $C(4, 0)$, $D(0, 0)$. Apply the scaling parameter 3 towards M axis and 4 towards N axis and obtain the new coordinates of the object.

Answer-

Given-

- Original coordinates of the square = A (0, 4), B(4, 4), C(4, 0), D(0, 0)
- Scaling factor along M axis = 3
- Scaling factor along N axis = 4

For Coordinates A(0, 4)

Applying the scaling equations, we have-

$$M' = M. S_M = 0 \times 3 = 0$$

$$N' = N. S_N = 4 \times 4 = 16$$

Thus, New coordinates of corner A after scaling = (0, 16).

For Coordinates A(4, 4)

Applying the scaling equations, we have-

$$M' = M. S_M = 4 \times 3 = 12$$

$$N' = N. S_N = 4 \times 4 = 16$$

Thus, New coordinates of corner A after scaling = (12, 16).

For Coordinates A(4, 0)

Applying the scaling equations, we have-

$$M' = M. S_M = 4 \times 3 = 12$$

$$N' = N. S_N = 4 \times 0 = 0$$

Thus, New coordinates of corner A after scaling = (12, 0).

For Coordinates A(0, 0)

Applying the scaling equations, we have-

$$M' = M. S_M = 0 \times 3 = 0$$

$$\mathbf{N}' = \mathbf{N} \cdot \mathbf{S}_N = 0 \times 4 = 0$$

Thus, New coordinates of corner A after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0,16), B(12, 16), C(12, 0), D(0, 0).

Rotation

Object is rotated θ about origin θ positive for counterclockwise otherwise negative.

The original coordinate P (M,N) as-

$$M = r \cos \phi$$

$$N = r \sin \phi$$

New co-ordinate point P' (M',N') as-

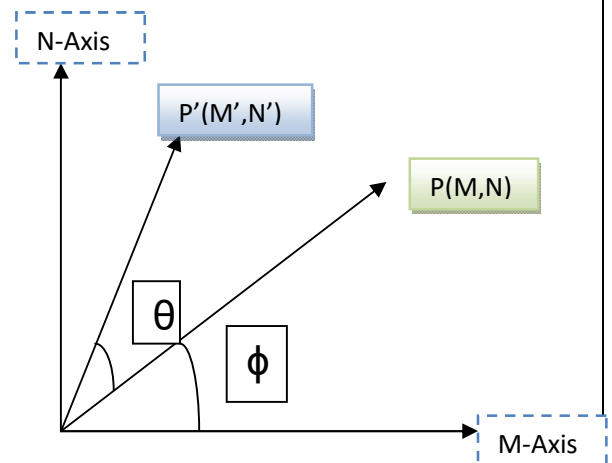
$$M' = r \cos(\phi + \theta) = r \cos \phi \cos \theta - r \sin \phi \sin \theta \dots (1)$$

$$N' = r \sin(\phi + \theta) = r \cos \phi \sin \theta + r \sin \phi \cos \theta \dots (2)$$

Calculate equation 3 and 4 , we will get

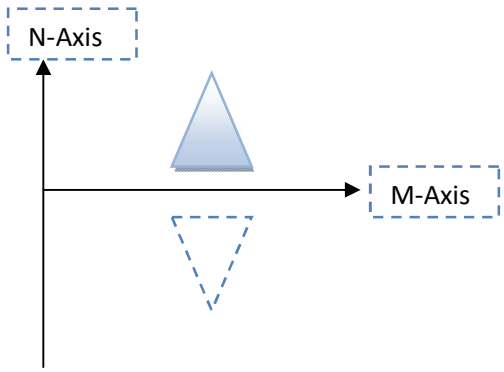
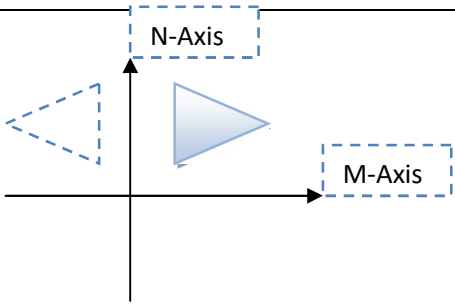
$$M' = M \cos \theta - N \sin \theta$$

$$N' = M \sin \theta + N \cos \theta$$



Mirror Reflection

- In this reflection the size of the object does not change.
- The mirror image can be either about M-axis or N-axis.

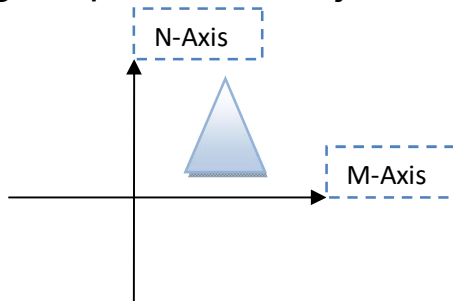
<p>About M-axis:</p> <p>Original point : M,N New reflected point : M',N'</p> $M'=M$ $N'=-N$	
<p>About N-axis:</p> <p>Original point : M,N New reflected point : M',N'</p> $M'=-M$ $N'=N$	

Composite Transformation

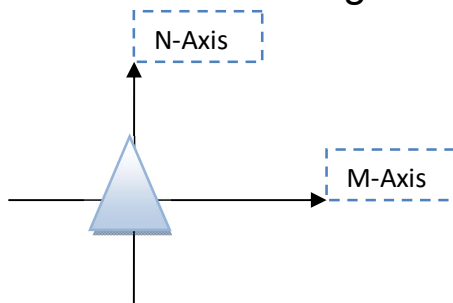
- Performing more than one transformation can be combined.
- This process of combining is called concatenation.
- Example : Suppose we want to perform rotation about an arbitrary point, then we can perform it by the sequence of three transformations
 1. Translation
 2. Rotation
 3. Reverse Translation

Example: the sequence of three transformations

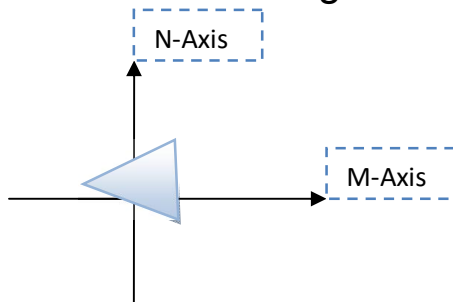
(a) Original position of object.



(b) Object Translate to origin.



(c) Object rotation to origin.



(d) Object retranslated to original position.

