Monad University,Hapur

Programme Name- B.Tech (ME)
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Subject Name- Applied Thermodynamics

Topic:- Thermodynamics Relation
Sub Topic:- Maxwell Equation

Thermodynamics Relation

Some Mathematics theorems:-

Theorem 1:- If a relation exists among the variables x,y and z, then z may be expressed as a function of x and y.

$dz= (\frac{\partial z}{\partial x})_y dx+(\frac{\partial z}{\partial y})_x dy$

if $(\frac{\partial z}{\partial x})_y$=M and $(\frac{\partial z}{\partial y})_x$=N, then dz=Mdx+Ndy is called exact differential equation. Where M & N are the function of x and y.

And $(\frac{\partial M}{\partial y})_x=(\frac{\partial N}{\partial x})_y$ ……… this is the condition of exact differential equation.

Theorem 2:- If a quantity f is a function of x,y and z and a relation exist among x,y and z , then f is a function of any two of the x, y and z. Similarly any one of x,y and z may be regarded to be a function of f and any one of x,y and z.

Thus, if x=x(f,y)

$dx=(\frac{\partial x}{\partial f})_y df+(\frac{\partial x}{\partial f})_f dy$

and $(\frac{\partial x}{\partial y})_f(\frac{\partial y}{\partial z})_f(\frac{\partial y}{\partial x})_f=1$

Theorem 3:- Among the variables x,y and z any one variable may be considered as a function of other two, thus

x=x(y,z)

$dx= (\frac{\partial x}{\partial y})_z dy+(\frac{\partial x}{\partial z})_y dz$

and $(\frac{\partial x}{\partial y})_z(\frac{\partial z}{\partial x})_y(\frac{\partial y}{\partial z})_x= -1$

Maxwell Equation

We know that from 1^{st} Law of thermodynamics

du=Q-Pdv ………..(a)

from theorem 1

If dz=Mdx+Ndy ………..(b), then $(\frac{\partial M}{\partial y})_x=(\frac{\partial N}{\partial x})_y$

After comparing equation (a) and (b)

$(\frac{\partial T}{\partial v})_s= -(\frac{\partial P}{\partial s})_v$ ………(1)

We know that from enthalpy equation

dH= U+PV

After differentiating above equation

dH= du+Pdv+vdP

dH= dQ+vdP

dH= Tds+vdP ……..(c)

After comparing equation (b) and (c)
$(\frac{\partial T}{\partial P})_s=(\frac{\partial v}{\partial s})_p$ ………(2)

From Helholtz function.
F=U-TS
After differentiating above equation
dF= dU-Tds-sdT
dF= -pdv-sdT ………….(d)
After comparing equation (b) and (d)
$-(\frac{\partial p}{\partial T})_v = -(\frac{\partial s}{\partial v})_T$

$(\frac{\partial p}{\partial T})_v = (\frac{\partial s}{\partial v})_T$…………..(3)

Fron Gibbs function :-
G=H-TS
After differentiating above equation
dG= dH- Tds-sdT
dG= vdp-sdT ……………(e)
After comparing equation (e) and equation (b)
$(\frac{\partial v}{\partial T})_p = -(\frac{\partial s}{\partial p})_T$ ……………..(4)

The above four equations (1),(2),(3) and (4) are called Maxwell Equation.

$(\frac{\partial T}{\partial v})_s= -(\frac{\partial P}{\partial s})_v$ ………(1)

$(\frac{\partial T}{\partial P})_s=(\frac{\partial v}{\partial s})_p$ ………..(2)

$(\frac{\partial p}{\partial T})_v = (\frac{\partial s}{\partial v})_T$………(3)

$(\frac{\partial v}{\partial T})_p = -(\frac{\partial s}{\partial p})_T$ ……..(4)